Unified Primal-Fractal Resonance Theory: Bridging Primordial Nucleosynthesis and Cosmic Expansion with Scale-Dependent Fractality

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1 Numerical Example: Evolution of the Fractal Scalar Field $\phi(t)$phi(t)

We consider the fractal scalar field $\phi(t)$ governed by the following equation of motion:

$$\ddot{\phi} + 3H(t)\dot{\phi} + m_\phi^2(\phi - f_{univ}) = 0, \tag{1}$$

where:

- $H(t)$ is the Hubble expansion rate,
- m_ϕ is the effective mass associated with the scalar field,
- $f_{univ} \approx 1.3745$ is the universal harmonic value linked to the fractal cosmic structure.

The goal of this section is to show that, due to the cosmological friction term $3H\dot{\phi}$, the energy associated with ϕ decreases sufficiently rapidly, ensuring that it does not disrupt the standard cosmological evolution at late times.

1.1 Assumptions and Simplifications

For this qualitative study:

- We model $H(t)$ as a simple decreasing function:

$$H(t) = \frac{h_0}{t}, \tag{2}$$

  where h_0 depends on the cosmological era:

  - During the Planck epoch: $h_0 \sim 1$,
  - During Big Bang Nucleosynthesis (BBN): $h_0 \sim 0.5 - 1$,

- At the current epoch: $H(t_0) \sim 2.3 \times 10^{-18}\, s^{-1}$.

- We take $m_\phi \approx 8.463 \times 10^{43}\, s^{-1}$ as derived previously.
- We neglect the backreaction of ϕ on $H(t)$, assuming ϕ is subdominant during the radiation and matter eras.

1.2 Early Evolution of $\phi(t)$phi(t)

Substituting $H(t) = h_0/t$ into Eq. (1) yields:

$$\ddot{\phi} + \frac{3h_0}{t}\dot{\phi} + m_\phi^2(\phi - f_{univ}) = 0. \tag{3}$$

At very early times ($t \to t_p \sim 10^{-43}\, s$), the friction term dominates because $1/t$ is large:

$$\ddot{\phi} \ll 3H\dot{\phi}.$$

Thus, the approximate equation becomes:

$$3H(t)\dot{\phi} + m_\phi^2(\phi - f_{univ}) \approx 0. \tag{4}$$

This leads to a slow evolution of $\phi(t)$ towards f_{univ}, heavily damped by the rapid cosmic expansion.

1.3 Transition to Damped Oscillations

As the Universe expands and t increases, $H(t)$ decreases. When $H(t) \sim m_\phi$, the scalar field begins to oscillate significantly around f_{univ}.

At this stage, Eq. (3) becomes that of a damped harmonic oscillator:

$$\phi(t) \approx f_{univ} + A(t)\sin(m_\phi t + \delta),$$

where the amplitude $A(t)$ decays due to cosmological friction.

In the asymptotic regime ($t \gg t_p$), the solution behaves as:

$$\phi(t) \approx f_{univ} + \frac{A_0}{t^{3h_0/2}}\sin(m_\phi t + \delta), \tag{5}$$

where A_0 is determined by initial conditions.

1.4 Energy Associated with ϕphi

The mean energy density associated with ϕ is:

$$\langle\rho_\phi\rangle = \frac{1}{2}\langle\dot{\phi}^2\rangle + \frac{1}{2}m_\phi^2\langle(\phi - f_{univ})^2\rangle. \tag{6}$$

Given that the amplitude decays as $t^{-3h_0/2}$, the energy density evolves as:

$$\langle \rho_\phi \rangle \propto t^{-3h_0}. \tag{7}$$

Thus, for $h_0 > 0.5$, the energy density of ϕ decreases rapidly with cosmic time.

1.5 Conclusion of the Numerical Example

In this minimalistic framework:

- At the earliest times ($t \sim t_p$), $\phi(t)$ is strongly damped and remains close to f_{univ}.
- During later epochs, $\phi(t)$ undergoes small, rapidly damped oscillations around f_{univ}.
- The associated energy density $\langle \rho_\phi \rangle$ decreases naturally as the Universe expands, avoiding any disruption to the standard cosmological evolution.

This confirms that the inclusion of a fractal scalar field $\phi(t)$ is physically consistent, provided the natural cosmological damping due to expansion is properly taken into account.