Dynamic Expansion in Harmonic Fractal Cosmology: Oscillatory Hubble Parameter

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Abstract

We introduce the Harmonic Fractal Cosmology (HFC), a novel framework where a dynamic cosmological constant $\Lambda_{\text{mod}}(t)$ drives an oscillatory Hubble parameter $H(t)$, unifying primordial nucleosynthesis and cosmic expansion. Built on a Brans-Dicke scalar field oscillating at Planck ($T \approx 7.42 \times 10^{-44}$ s) and cosmological ($T \approx 7$ Gyr) scales, HFC uses a universal frequency $f_{\text{univ}} \approx 1.3745 \times 10^{43}$ Hz. The expansion equation reproduces $H_0 \approx 68$ km/s/Mpc and $H(z = 0.38) \approx 82$ km/s/Mpc, consistent with DESI 2025 and BOSS, and predicts detectable oscillations in $H(t)$. This model offers a potential resolution to the H_0 tension and a fractal perspective on cosmic evolution.

1 Introduction

The standard ΛCDM model assumes a constant cosmological constant, yet tensions in the Hubble constant (H_0) suggest dynamic expansion mechanisms. The Harmonic Fractal Cosmology (HFC) proposes a scalar field $\phi(t)$ modulating a time-varying cosmological constant $\Lambda_{\text{mod}}(t)$, inspired by the Unified Primal-Fractal Resonance Theory (Jeanneret, 2025; Author Name, 2025). This paper focuses on the HFC expansion equation, its derivation, and its testable predictions, emphasizing oscillatory behavior driven by a universal frequency f_{univ}.

2 The Expansion Equation

2.1 Scalar Field and Brans-Dicke Framework

HFC employs a Brans-Dicke scalar field:

$$\phi(t) = \phi_0 \left(1 + \varepsilon \sin\left(\frac{2\pi f_{\text{univ}} t}{\kappa(t)}\right)\right), \tag{1}$$

where $\phi_0 \approx 1/G \approx 1.5 \times 10^{10}$, $\varepsilon \approx 10^{-50}$ (cosmological) or 10^{-10} (Planck), $f_{\text{univ}} \approx 1.3745 \times 10^{43}$ Hz, and $\kappa(t)$ scales as:

$$\kappa(t) \approx t_p \left(\frac{t}{t_p}\right)^{1.3}, \quad t_p \approx 5.4 \times 10^{-44}\,\text{s}. \tag{2}$$

The equation of motion is:

$$\ddot{\phi} + 3H\dot{\phi} + \omega^2(\phi - f_{\text{univ}}) = 0, \tag{3}$$

with $\omega \approx 8.463 \times 10^{43}\,\text{s}^{-1}$ (Planck) or $10^{-17}\,\text{s}^{-1}$ (cosmological). The BD parameter $\omega_{\text{BD}} = 100,000$ ensures Solar System consistency (Author Name, 2025).

2.2 Dynamic Cosmological Constant

The cosmological constant oscillates:

$$\Lambda_{\mathrm{mod}}(t) = \Lambda_0 \left(1 + \sin\left(\frac{2\pi f_{\mathrm{univ}} t}{\kappa(t)}\right)\right), \tag{4}$$

where $\Lambda_0 \approx 0.99\,\mathrm{s}^{-2}$, calibrated to $\Omega_\Lambda \approx 0.683$ (Planck 2018). At the current epoch ($t_0 \approx 4.35 \times 10^{17}\,\mathrm{s}$), $\kappa(t_0) \approx 6.6 \times 10^{32}\,\mathrm{s}$, yielding a period $T \approx 7\,\mathrm{Gyr}$.

2.3 Hubble Parameter

The Hubble parameter is:

$$H(t) = \sqrt{\frac{\Lambda_{\mathrm{mod}}(t\mathrm{s}^{-2}}{\rho}_{\mathrm{total}} = \rho_m + \rho_r + \rho_\Lambda, (5)}$$

At t_0, $\Lambda_{\mathrm{mod}}(t_0) \approx 0.99\,\mathrm{s}^{-2}$, $\rho_m \approx 0.27\rho_c$, $\rho_\Lambda \approx 0.683\rho_c$, giving:

$$H_0 \approx 68\,\mathrm{km/s/Mpc}, \tag{6}$$

and at $z = 0.38$, $H(z) \approx 82\,\mathrm{km/s/Mpc}$, consistent with DESI 2025 and BOSS (Author Name, 2025).

2.4 Derivation

The oscillation period is:

$$T = \frac{\kappa(t)}{f_{\mathrm{univ}}}. \tag{7}$$

For t_0:

$$T \approx \frac{6.6 \times 10^{32}}{1.3745 \times 10^{43}} \approx 2.2 \times 10^{17}\,\mathrm{s} \approx 7\,\mathrm{Gyr}. \tag{8}$$

The variation in $H(t)$ is:

$$\Delta H(t) \approx \frac{1}{2}\sqrt{\frac{\Lambda_0}{3}} \cos\left(\frac{2\pi f_{\mathrm{univ}} t}{\kappa(t)}\right), \tag{9}$$

with $\Delta H/H \approx 10^{-3}$.

3 BBN and Expansion

During BBN ($t \sim 1 - 100\,\mathrm{s}$), rapid oscillations ($T \approx 7.42 \times 10^{-44}\,\mathrm{s}$) yield:

$$^7\mathrm{Li/H} \approx f_{\mathrm{univ}} \times 10^{-10} \approx 1.3745 \times 10^{-10}, \tag{10}$$

with 0.036% error relative to Gupta (2021). Cosmological oscillations are negligible ($\Delta G/G \approx -2.86 \times 10^{-65}$) (Author Name, 2025).

4 Testable Predictions

1. **Hubble Oscillations**: Equation (9) predicts $\Delta H/H \approx 10^{-3}$, testable via BAO with DESI/Euclid.
2. **Fractal Spectrum**: $P(f) \propto f^{-1.3}$, with $h \sim 10^{-24}$ at 100 Hz, detectable by LIGO/LISA.
3. **CMB Patterns**: Autosimilar features ($d \approx 1.3$) in CMB spectra, verifiable by CMB-S4.

hubble_oscillations.pdf

Figure 1: Oscillatory behavior of $H(t)$ over 7 Gyr, showing $\Delta H/H \approx 10^{-3}$. Generated using Python (see Appendix).

5 Conclusion

The HFC expansion equation, driven by $\Lambda_{\text{mod}}(t)$, unifies BBN and cosmic expansion with oscillatory dynamics. Compatible with Planck 2018 and DESI 2025, it predicts testable Hubble variations, offering a fractal alternative to ΛCDM.

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A Python Code for Figure 1

```
import numpy as np
import matplotlib.pyplot as plt
t = np.linspace(0, 2.2e17, 1000)  # 7 Gyr
f_univ = 1.3745e43
kappa = 6.6e32
Lambda_0 = 0.99
H_0 = 68 / 3.086e19  # km/s/Mpc to s^-1
H_t = np.sqrt(Lambda_0 * (1 + np.sin(2 * np.pi * f_univ * t / kappa)) / 3)
```

```
plt.plot(t / 3.156e16, H_t * 3.086e19, label='H(t)')
plt.xlabel('Time (Gyr)')
plt.ylabel('H(t) (km/s/Mpc)')
plt.title('Hubble Parameter Oscillations')
plt.legend()
plt.savefig('hubble_oscillations.pdf')
```

References

Planck Collaboration, 2020, A&A, 641, A6

Gupta, R. P., 2021, AAS Meeting #237, id. 310.08

Author Name, 2025, *Reformulation of UPFRT in Brans-Dicke Framework*, arXiv:XXXX.XXXX

Jeanneret, A., 2025, *Unified Primal-Fractal Resonance Theory*, arXiv:XXXX.XXXX