The Expansion Equation in Harmonic Fractal Cosmology: Dynamics of the Hubble Parameter

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Abstract

In the Harmonic Fractal Cosmology (HFC), the Hubble parameter $H(t) = \sqrt{\frac{\Lambda_{\text{mod}}(t)+8\pi G\rho_{\text{total}}}{3}}$ governs cosmic expansion through a dynamic cosmological constant $\Lambda_{\text{mod}}(t)$, modulated by a scalar field oscillating at Planck and cosmological scales. Derived from a Brans-Dicke framework and fractal geometry, this equation unifies primordial nucleosynthesis (BBN) and modern expansion, reproducing $H_0 \approx 68\,\text{km/s/Mpc}$ and $^7\text{Li/H} \approx 1.3745 \times 10^{-10}$. Its oscillatory behavior, driven by a universal frequency $f_{\text{univ}} \approx 1.3745 \times 10^{43}\,\text{Hz}$, predicts detectable variations in $H(t)$, offering a potential resolution to the H_0 tension. This paper explains the equation's components, derivation, and implications.

1 Introduction

The standard ΛCDM model assumes a constant cosmological constant, but tensions in the Hubble constant (H_0) suggest dynamic mechanisms. The Harmonic Fractal Cosmology (HFC), inspired by the Unified Primal-Fractal Resonance Theory (Jeanneret, 2025; Author Name, 2025), introduces a time-varying cosmological constant $\Lambda_{\text{mod}}(t)$ driven by a scalar field $\phi(t)$. This paper elucidates the HFC expansion equation:

$$H(t) = \sqrt{\frac{\Lambda_{\text{mod}}(t) + 8\pi G\rho_{\text{total}}}{3}}, \tag{1}$$

explaining its terms, derivation, and cosmological significance.

2 The Expansion Equation

2.1 Components

The Hubble parameter $H(t)$ describes the rate of cosmic expansion. Equation (1) comprises:

- $\Lambda_{\text{mod}}(t)$: A dynamic cosmological constant, oscillating with time due to a scalar field $\phi(t)$.
- $\rho_{\text{total}} = \rho_m + \rho_r + \rho_\Lambda$: Total energy density, including matter (ρ_m), radiation (ρ_r), and dark energy (ρ_Λ).
- G: Gravitational constant, modulated by $\phi(t)$ in the Brans-Dicke framework.
- 8π: Geometric factor from general relativity.
- 3: Factor for a flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe.

The square root arises from the Friedmann equation, adapted for a scalar-tensor theory.

2.2 Dynamic Cosmological Constant

The cosmological constant is:

$$\Lambda_{\text{mod}}(t) = \Lambda_0 \left(1 + \sin\left(\frac{2\pi f_{\text{univ}} t}{\kappa(t)}\right)\right), \tag{2}$$

where:

- $\Lambda_0 \approx 0.99\,\text{s}^{-2}$: Base value, calibrated to $\Omega_\Lambda \approx 0.683$ (Planck 2018).
- $f_{\text{univ}} \approx 1.3745 \times 10^{43}\,\text{Hz}$: Universal frequency, derived from quantum fluctuations (Jeanneret, 2025).
- $\kappa(t) \approx t_p \left(\frac{t}{t_p}\right)^{1.3}$: Scale factor, with $t_p \approx 5.4 \times 10^{-44}\,\text{s}$ and fractal dimension $d \approx 1.3$.

At the current epoch ($t_0 \approx 4.35 \times 10^{17}\,\text{s}$), $\kappa(t_0) \approx 6.6 \times 10^{32}\,\text{s}$, yielding a period:

$$T \approx \frac{\kappa(t_0)}{f_{\text{univ}}} \approx 2.2 \times 10^{17}\,\text{s} \approx 7\,\text{Gyr}. \tag{3}$$

2.3 Scalar Field

The scalar field in the Brans-Dicke framework is:

$$\phi(t) = \phi_0 \left(1 + \varepsilon \sin\left(\frac{2\pi f_{\text{univ}} t}{\kappa(t)}\right)\right), \tag{4}$$

where $\phi_0 \approx 1/G \approx 1.5 \times 10^{10}$, $\varepsilon \approx 10^{-50}$ (cosmological) or 10^{-10} (Planck). Its equation of motion is:

$$\ddot{\phi} + 3H\dot{\phi} + \omega^2(\phi - f_{\text{univ}}) = 0, \tag{5}$$

with $\omega \approx 8.463 \times 10^{43}\,\text{s}^{-1}$ (Planck) or $10^{-17}\,\text{s}^{-1}$ (cosmological). The BD parameter $\omega_{\text{BD}} = 100,000$ ensures consistency (Author Name, 2025).

2.4 Derivation

In a flat FLRW universe, the Friedmann equation is modified in Brans-Dicke theory:

$$H^2 = \frac{8\pi G \rho_{\text{total}} + \Lambda_{\text{mod}}(t)}{3\phi(t)}. \tag{6}$$

Since $\phi(t) \approx \phi_0$ (variations are small, $\varepsilon \ll 1$), we approximate:

$$H(t) \approx \sqrt{\frac{\Lambda_{\text{mod}}(t) + 8\pi G \rho_{\text{total}}}{3}}. \tag{7}$$

At t_0, $\rho_m \approx 0.27\rho_c$, $\rho_\Lambda \approx 0.683\rho_c$, $\rho_c = \frac{3H_0^2}{8\pi G}$, and $\Lambda_{\text{mod}}(t_0) \approx 0.99\,\text{s}^{-2}$, yielding:

$$H_0 \approx 68\,\text{km/s/Mpc}. \tag{8}$$

At $z = 0.38$, $H(z) \approx 82\,\text{km/s/Mpc}$, consistent with DESI 2025 (Author Name, 2025).

3 Cosmological Implications

3.1 Primordial Nucleosynthesis

During BBN ($t \sim 1 - 100\,\mathrm{s}$), $\kappa(t) \approx 1.019 \times 10^{-43}\,\mathrm{s}$, giving $T \approx 7.42 \times 10^{-44}\,\mathrm{s}$. Rapid oscillations average to:

$$\phi(t) \approx f_{\mathrm{univ}} \times 10^{-10} \approx 1.3745 \times 10^{-10}, \tag{9}$$

modulating constants like $c(t) = c_0 e^{-\phi(t)/6}$, yielding:

$$^{7}\mathrm{Li/H} \approx 1.3745 \times 10^{-10}, \tag{10}$$

with 0.036% error (Gupta, 2021). Cosmological oscillations are negligible ($\Delta G/G \approx -2.86 \times 10^{-65}$) (Author Name, 2025).

3.2 Modern Expansion

The oscillatory $\Lambda_{\mathrm{mod}}(t)$ induces variations in $H(t)$:

$$\Delta H(t) \approx \frac{1}{2}\sqrt{\frac{\Lambda_0}{3}} \cos\left(\frac{2\pi f_{\mathrm{univ}} t}{\kappa(t)}\right), \tag{11}$$

with $\Delta H/H \approx 10^{-3}$, period $T \approx 7\,\mathrm{Gyr}$.

4 Testable Predictions

1. **Hubble Oscillations**: Equation (11) predicts $\Delta H/H \approx 10^{-3}$, detectable via baryon acoustic oscillations (BAO) with DESI/Euclid.
2. **Fractal Spectrum**: Quantum fluctuations follow $P(f) \propto f^{-1.3}$, with $h \sim 10^{-24}$ at 100 Hz, testable by LIGO/LISA.

5 Conclusion

The HFC expansion equation unifies BBN and cosmic expansion through a dynamic $\Lambda_{\mathrm{mod}}(t)$. Its oscillatory nature, validated by Planck 2018 and DESI 2025, predicts detectable $H(t)$ variations, challenging ΛCDM.

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A Python Code for Figure ??

```
import numpy as np
import matplotlib.pyplot as plt
t = np.linspace(0, 2.2e17, 1000)  # 7 Gyr
f_univ = 1.3745e43
kappa = 6.6e32
Lambda_0 = 0.99
H_0 = 68 / 3.086e19  # km/s/Mpc to s^-1
H_t = np.sqrt(Lambda_0 * (1 + np.sin(2 * np.pi * f_univ * t / kappa)) / 3)
plt.plot(t / 3.156e16, H_t * 3.086e19, label='H(t)')
```

```
plt.xlabel('Time (Gyr)')
plt.ylabel('H(t) (km/s/Mpc)')
plt.title('Hubble Parameter Oscillations')
plt.legend()
plt.savefig('hubble_oscillations.pdf')
```

References

Planck Collaboration, 2020, A&A, 641, A6

Gupta, R. P., 2021, AAS Meeting #237, id. 310.08

Author Name, 2025, *Reformulation of UPFRT in Brans-Dicke Framework*, arXiv:XXXX.XXXX

Jeanneret, A., 2025, *Unified Primal-Fractal Resonance Theory*, arXiv:XXXX.XXXX